A NEW ALGORITHM FOR THE CITY: THE USE OF TOPOLOGY AND TRANSPORT MODELING TO MAKE URBAN AREAS MORE EQUITABLE

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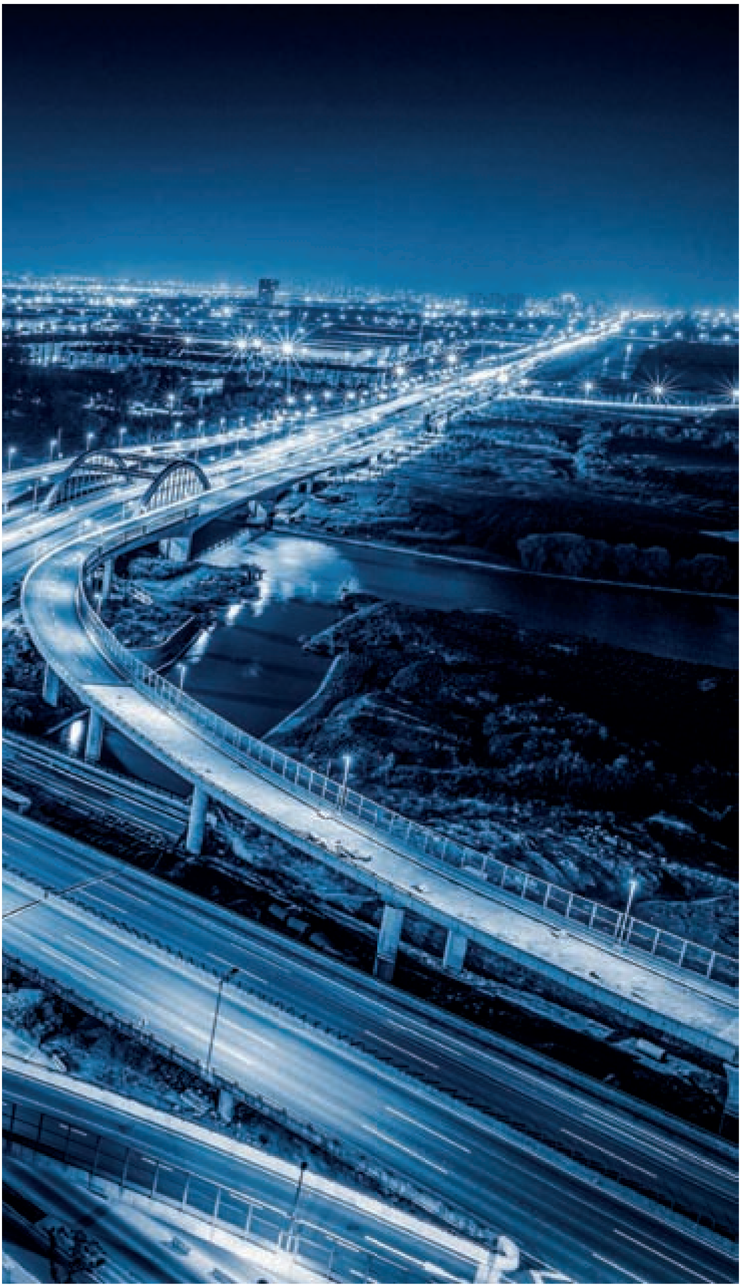
The prevailing model of urban development is increasingly oriented to a territorial isotropy, despite the evidences of a considerable literature.



This is due to the lack of mathematical models able to translate this awareness into appropriate analytical and functional models.
Nowadays a city no longer exists as mere physical built place, but as a dematerialized "cloud" of flows: people, goods, information. The paper presents a methodology that allows us to use the modeling tools already in the planning and design phases to promote new mobility paradigms that reduce the displacement need and the soil consumption.
The concept of Set is the cornerstone of most of modern mathematical expositions. It is primitive and intuitive, because it is introduced as a generalization of the concept of a finite set, like a box containing homogeneous material objects. A domain is the closure of an open set, i.e. a closed set formed by the union between an open set and its border: a neighborhood unit is a domain.
The proposal is to use the topological theory to highlight the physiological processes of a city. Through the measurement of the mileage it is possible identify the effective improvements: a functional "mixité" of a neighborhood unit; a connection of two clusters to reduce the radial displacements; a more homogeneous distribution of facilities. Proceeding in a controllable and measurable way.
Final aim is to promote a greater local distribution of the flows that make the slow mobility functional and structural.

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Introduction

Although this is the information age, it does not mean that the displacements are decreasing. The cities are being renovated but the per capita movement of people and goods will increase by 30% at 2050 (The World Bank). This means that a proper planning of displacements will be essential for achieve the sustainability. The guiding principle is that a balanced city produces social - and environmental – well-being.

To make possible a restructuring of urban flows, it is necessary to rethink the relationship between mobility and the isotropic structure of the contemporary territory. In fact, whether the dispersion is considered as a deleterious phenomenon to be stopped, or it is accepted as a positive process of the current phase of capitalism, the spatial results of 40 years of dispersion process are almost irreversible.

It is necessary to design a mobility system that mitigates the inefficiencies and diseconomies of real estate industry, without denying the specific dilated conformation of the dispersion. An integrated system able to reorganize what exists but that is able to evolve over time. This requires reviewing the philosophy underlying the conceiving of transport networks, that is to rethink the relationship between the consolidated city and sprawl.

Today to a territorial isotropy that expands the city up to coalesce into a regional-scale urbanized continuum – corresponds an anachronistic logic that sees a centripetal transport planning from downtown to the suburbs. In this frame, the transport modelling is involved at the end of urban planning, when all decisions have been already made. This is because the mathematical modeling is perceived as a verification tool, at most as a tool useful to sizing the urban items.

The paper presents a methodology that allows us to use the modeling tools already in the planning and design phases to promote new mobility paradigms that reduce the displacements. The measurement unit is the mileage, i.e. the number of trips by the traveled distance. The urban body is discretized in homogeneous areas, the neighborhood units. More neighborhood units form urban clusters. In carrying out their daily activities, people will choose to move into or out of a cluster, depending on whether they can find what they need in the same cluster.

Territorial division should follow a careful reading of the territory: this means translating the physical elements in a mathematical language. Area, border, barrier: these are all concepts describable by the principles of the Analytical topology. The concept of Set is the cornerstone of most of modern mathematical expositions. It is a primitive element, because it is introduced as a concept not derivable from more basic concepts. It is intuitive, because it is introduced as a generalization of the concept of a finite set, like a box containing homogeneous material objects. A domain is the closure of an open set, i.e. a closed set formed by the union between an open set and its border: a neighborhood unit is a domain.

The topological model

The proposal is to use the topological theory to highlight the physiological processes of a city. Through the measurement of the mileage it is possible identify the needed improvements: a functional “mixité” of a neighborhood unit; a connection of two clusters to reduce the radial displacements; a more homogeneous distribution of new facilities.

All to promote a greater local distribution of the flows that make the slow mobility functional and structural.

In mathematics, a topological space is the basic object of Topology. It is a very general concept of space, accompanied by a notion of "closeness" defined as weak as possible. Thus, many of the spaces commonly used in mathematics (such as Euclidean geometric space) are nothing more than topological spaces. What virtually characterizes a topological space is its form, and not the distance between its points. this is because the distance between two points X and Y cannot be defined as a general law using topology rules.

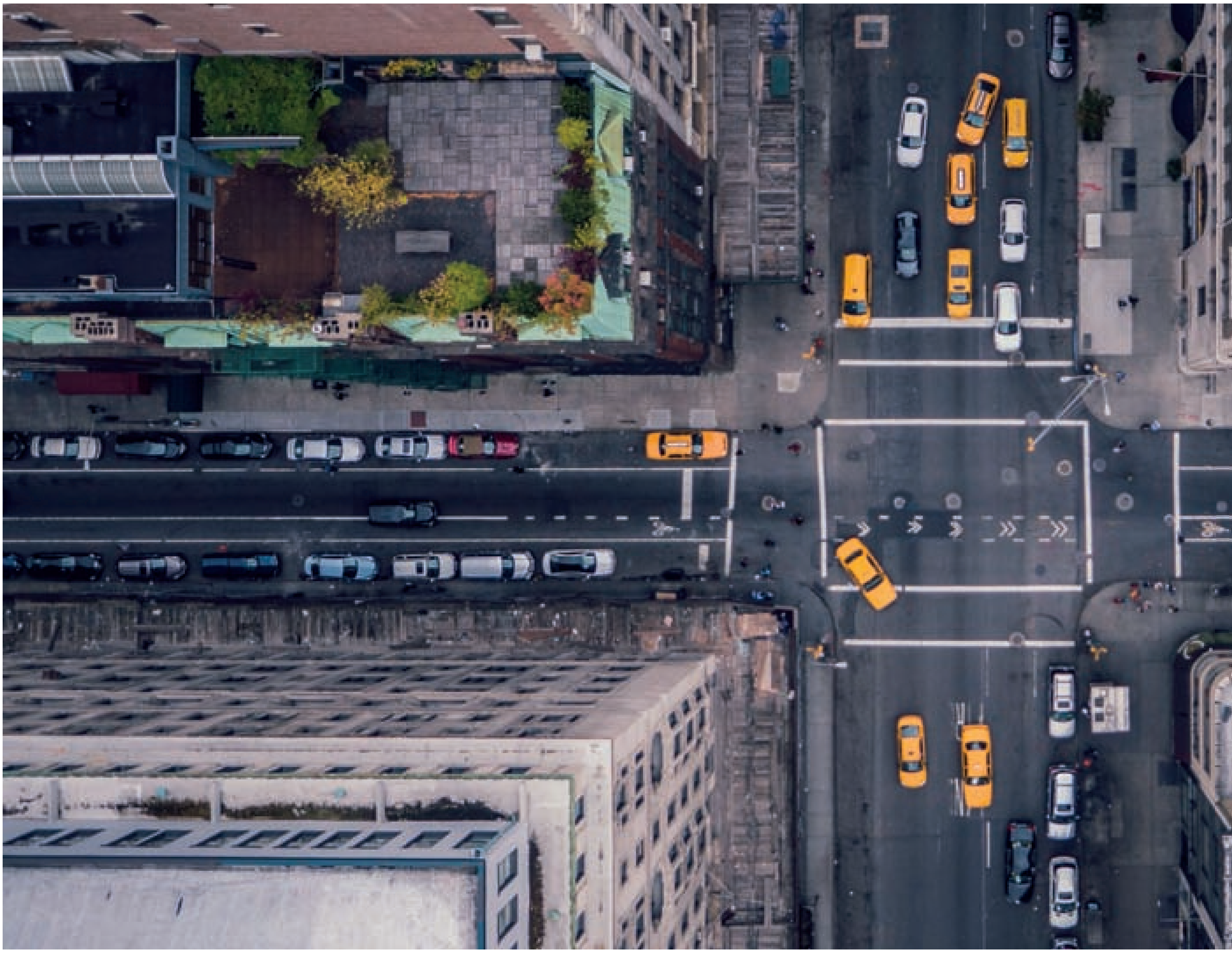
The topology is one of the most important branch-

> **The proposal is to use the topological theory to highlight the physiological processes of a city.**

es of modern mathematics: here fundamental concepts such as convergence, limits, continuity, connection or compactness found their best formalization.

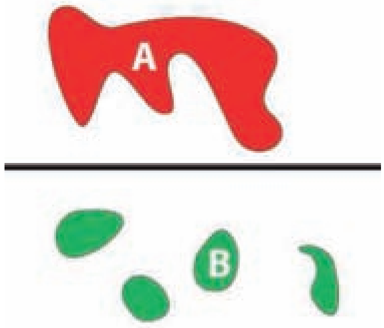
The concept of closure is found in mathematics in many fields and with different degrees of generality. Intuitively, a set is close if it is possible to move enough in any direction from any point in the set without departing from the collection itself. Following the general definitions one can move beyond this intuitive idea. Through the definition of open set, it is possible to define terms like "near", "far", "attached", "separate". Non-intuitive definition of open sets corresponds to mathematical situations in which these concepts are used in a non-intuitive way. It is the case, for example, of non-Euclidean geometry.

A domain is connected if it is not divisible into in-



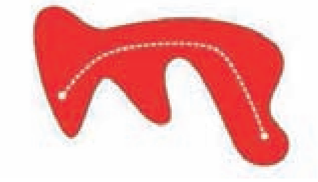
dependent portions. In mathematical terms is said that a nonempty topological space is called connected if the only pair of disjoint open subsets (separated) whose union is X are {Ø, X}. These are called banal subsets: the empty one and the X itself.
In simpler words - but quite intuitive - it can be said that the connection is the topological property of a set of being formed by a single "piece".
The limit connected subsets of a topological space X are the connected components of X. In other words, a subset of X is a connected component if it is connected and is not contained in any other subset connected. The connected components of X are disjoint and their union is X: they form a partition of X. In most problems it is sufficient to consider only connected space, because they are the "building blocks" in which are made all topological spaces.

Figure 1: "A" represent a connected domain while "B" not, it consists of 4 related - but disjointed – entities.



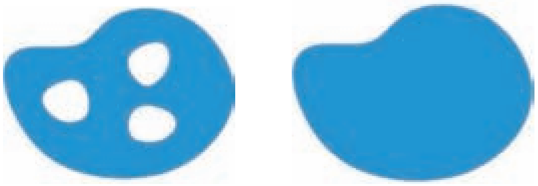
A topological space X is connected by arcs if for any two points x and y of the space there is a path that connects them. In other words, there is a continuous function f from the range [0, 1] in X so that f (0) = x and f (1) = y. A space connected by arcs is connected. The two notions do not coincide, because there are spaces connected but not connected by arcs.

Figure 2: Domain connected by a path.



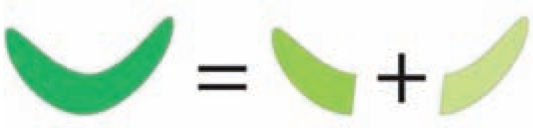
A domain is simply connected if it is made of one piece and has no holes. More precisely, a topological space is simply connected if is connected by arcs (one piece) and every closed curve can be deformed to be reduced to a single point (no holes). Is p a point of a domain X. A path starting from p is a continuous function f: [0, 1] → X such that f(0) = f(1) = p. The path is contractible if there exists a homology that transforms the path into the constant path g(t) = p for every t belonging to X. In other words, it is contractible if it can be "squeezed" continuously up to become arbitrarily small. The topological space X is simply connected if every path centered at p is contractible.

Figure 3: The domain on the left is not simply connected. It becomes connected by eliminating the holes (figure on the right).



In the Euclidean plane a Set is said convex if - for each pair of points (a, b) of the set - the segment ab that joins them is entirely contained in the set. Examples of convex sets are circles, spheres, cubes, planes, half planes, trapezoids. Not convex sets (called concave) are arcs of circles, torus or any collection containing holes or depressions. Generally, all the not connected sets are convex. Intuitively, a convex shape is a compact, while a concave figure is an arched shape.
The definition of concave set in not used in Set theory, but it is employed through the more complex notion of connected space.

Figure 4: Decomposition of a concave set in two convex subsets.



In the Euclidean space a convex set is simply connected. Vice versa a set concave can be divided into two or more convex subsets.

From the city to the archipelago of urban islands

The discretization of an urban area can be done in very different ways [1]. The proposed method involves the identification of homogeneous areas by territorial density. These elements are urban islands characterized by a degree of urbanistic homogeneity. Each island is a geographical space having the following characteristics:

- homogeneous urban structure, described by the homogeneity of territorial distribution;
- be topologically representable by a simply connected domain (that is a connected-for-density domain).

  Starting from a certain neighborhood – the pivot -, to delimitate the urban island it is possible to proceed in two ways:
- by aggregating neighboring entities to the pivot;
- by disaggregating administrative entities and then by aggregating to the pivot only the equipollent urban portions***.

Of course this is a simplified discourse. The density threshold value approximates a real situation that is much more complex. The urban structures are organized in parties related by multiple elements. The population density (conceived as an approximation of the contiguity of the built-up area) is just one of these elements. A city exists not as a fixed entity (built-up area) but as a cloud of high-density relational paths. People, goods and information move by following a complex logic but a robust literature [2] has shown that this complexity is related to residential density. A relationship that does not depend so much to the absolute value of the density, as far as the mutual differences between the various parts of the city.

Thanks to GIS tools [3], it is possible to automate the procedure described above by implementing the following algorithm:

1. acquire the administrative boundary of the municipality
2. acquire the territorial density values, at the most disaggregated value (e.g. urban districts or neighborhoods)
3. divide the municipal area according to an elemental grid of 250 meters' side
4. n = number of grid columns
5. m = number of grid rows
6. v = 1
7. w = 1
8. i = 1
9. for v = 1 to n repeat the following cycle:

Figure 5: A simplified urban area in which four distinct homogenous urban islands are discernible. The first, U1, is derived from the aggregation of three neighborhoods with different densities: the pivot area (dark colored) exceeds 3,000 people per km^2. The second urban area, U2, is formed by the incorporation of two neighborhoods for density and annexation of a third (with a density below the reference) by contiguity. U3 and U4 are mononuclear or banal.

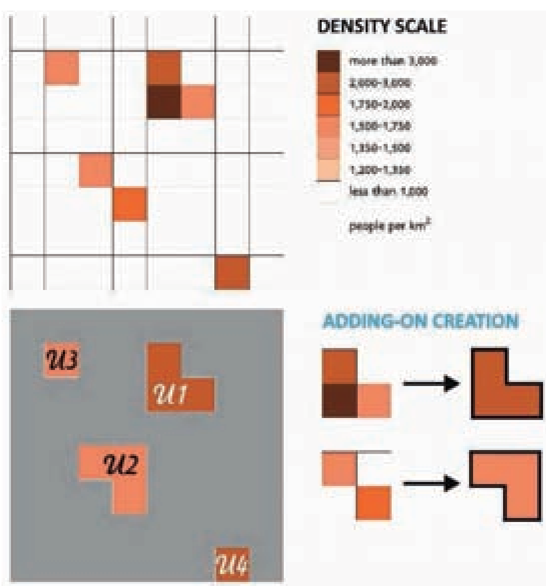
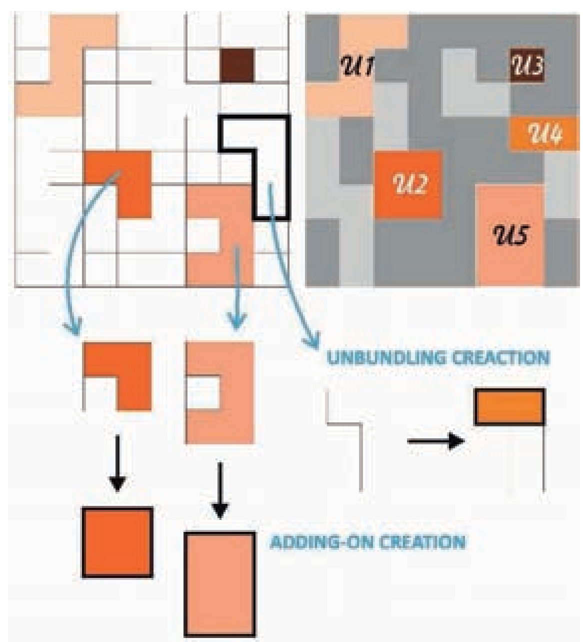


Figure 6: In the second example beside, it is shown the urbanism island be formed even for disaggregation of a demographically not homogeneous neighborhoods. Starting from a common non-simply connected domain, it is split into two sub-domains in which one hosts the main urban nucleus and the other the remaining land.



10. for w = 1 to m repeat the following cycle:
    11. domain assignment $U_i = c_{vw}$
    12. assign s_i = 1
    13. assign $h = n$
    14. assign $k = m$
    15. for i=1 to h repeat the following cycle:
        16. for j=1 to k repeat the following cycle:
            17. if $0{,}8 \cdot D_{hk} < D_{ij} < 1{,}2\ D_{hk}$
                then $U_i = U_i + c_{ij}$
                    and $j = j+1$
                    and $s_i = s_i+1$
                else $j=k$
            18. If s – i = 0 then $i = i + 1$
                else $i = h$
    19. $w = w + 1$
20. $v = v + 1$
21. end

The elementary function D_{hk} returns the territorial density value in the cell c_{hk}. The working grid has 250 meters' side because the distance of 250 meters can be assumed to be acceptable distance for most the population to be walked on foot in their daily trips.
In this way, the municipality inscribed in the grid $m \cdot n$, is divided into i urban islands U_i formed by k rows and s_i contiguous columns.
The next step is to make the urban islands U_i a topologically connected domains. The routine to apply is described by the following algorithm:
1. domain assignment $D = U_i$
2. if D is connected (in a topological meaning)
then go to end
else
3. D=D + internal holes
4. h = row (U_i)
5. k = column (U_i)
6. while D is not connected (in a topological meaning)
    7. if k<m-1 then k=k+1
       else h=h+1 and k = column (U_i)
    8. D=D+U_i (h,k)
9. end

This operation involves that the urban islands may be reduced by number since some of them might merge one each other to comply with the connection condition.
The discretization procedure in a set of urban islands topologically connected is applicable:
- to an administrative unit (municipality);
- to an urban area formed by multiple administrative units (larger urban zone or metropolitan area).

The trip generation model

For an urban transport line with N stations, the number of daily passengers [4] will be the result of two separate contributions:

Daily passenger = P_d = node effect+network effect

Denoting with S_i the i-th station or stop of the line:

$$P_d = \sum_{h=1}^{h=N} P_d(S_k) + \prod_{h=1}^{h=N} \prod_{k=1}^{k=N} P_d(S_h \leftrightarrow S_k)$$

The node effect is due to passengers generated or attracted by a specific single station, valued as if it were isolated and not inserted in the wide context.
The net effect is additional to the previous and results from the mutual synergies of the network: a passenger moves from A to C because he is interested to go to B.
In general, these modes of displacement are a function of the same elements in other words trip way = f (age class, average income, land services).
The model formulated on these rules, acts like a gravitational field [5]: each node exercises on a hypothetical user an action directly proportional to its population size and its equipment of services and inversely proportional to the distance that separates it from the own place of the user.
What was said above is also generalizable for private transport. In this case, the infrastructure on which passengers move is the urban road network. Stations and stops are the junctions and the car parks which are distributed along the road network.
In Italy (Istat), in an urban area it can be assumed that about 82% (it was 85% at the beginning of the 2000s) of the resident population make at least one trip every weekday. The number of trips made by people traveling in an urban area is 2.7.
For each urban island U_i of a city, with a population Πi, the number of daily generated trips T_i is

$$T_i = 0{,}85 \cdot \Pi_i \cdot 2.7$$

Currently urban planning requires to the transport modeling to improve the networks performance that is lower overall impedance. Road impedance from a node N_h to a node N_k is defined as

$$Z^R_{hk} = 1 + \alpha \cdot ln(\tau_{hk})$$

Where _(hk) is the travel time from h to k, and is an integer constant. τ_{hk} is a function of the average speed σ_{hk}

$$\sigma_{hk} = \sigma_{MAX} - \gamma_{hk}$$

γ_{hk} is the average congestion factor on the path hk. But also the following relationship is valid, where β is an integer constant

$$\gamma_{hk} = 1 + \beta \cdot ln(\Pi_i)$$

This means that road congestion is related to the population or that the reduction in traffic depends on the location of the attractors. Attractors which are the land services that to meet the needs of the population. Schools, hospitals, shops and shopping malls as well as jobs.

Of all daily trips T_i generated by an urban island U_i, a part I_i will remain within the perimeter of the island and another part O_i will go to the outside.

$$T_i = I_i + O_i = \iota \cdot T_i + o \cdot T_i$$
$$(1 - o) \cdot T_i + o \cdot T_i$$

In each case, the distances will be different so the impact on the city will be different. For the coefficient of the outside trips is $0<o<1$.

Containing the coefficient o means reducing the number of trips directed towards the outside of the urban island, i.e. reducing the overall mileage of daily urban trips. Reducing this ratio means balancing the mutual relations between the urban islands i.e. make each island U_i more attractive compared to the needs of its residents.

It is possible to set the following classes between the value of the coefficient and the character of an urban island

$0 \le o < 15\%$	closed
$15 \le o < 30\%$	moderately open
$30 \le o < 45\%$	open
$o \ge 45\%$	very open

A closed urban island is very rich in land services, on the contrary an open island is very poor in land services, and it forces its residents to move out.

Taking into account the physiological synergies within the different parts of an urban area (center, suburbs, fashionable quarters), it would be appropriate that each urban islands is at least moderately open, i.e. the outside trip ratio is $15 \le o < 30\%$.

Reprogramming the city

Discretizing a city - or an urban area - in a series of topologically connected element, allows to study more effectively the circadian physiology.

GIS tools allow to work on a large scale with a large amount of information. Big data allows a much more detailed reading of the flows than fieldwork's survey.

Floating car data (FCD), also known as floating cellular data, is a method to determine the traffic speed on the road network. It is based on the collection of localization data, speed, direction of travel and time information from mobile phones in vehicles that are being driven. Based on these data, traffic congestion can be identified, travel times can be calculated, and traffic reports can be rapidly generated. In contrast to traffic cameras, number plate recognition systems, and induction loops embedded in the roadway, no additional hardware on the road network is necessary.

For what concern public transport, the Automatic Vehicle Monitoring System (AVM) is GPS-based vehicle location information in real time, showing that information on a map. Further, it correlates vehicle information with customer-specific information and streamlines exchange of information between a dispatcher and vehicles.

FCD and AVM allows a real-time picture of urban flows, but also to weigh the effects of urban planning actions.

It is always more important not so much to reduce trips in their number as in their length. This means redesigning entire quarters, favoring slow-moving short-range mobility.

In this way the concept of urban island described above can become to plan environmental islands where the environment is conjugated also as the quality of social relationships.

Long distance trips, outside the urban islands, must be largely sustained by mass transport.

The assessment of a rapid transit line health ef-

fects must be related to local conditions of the project corridor. The evaluation [6] requires the following characteristic data:

- Average daily PM10 concentration (in g/m3)
- Traffic average PM10 emission rate (R)
- Daily road trip (RT)
- Daily road mileage (RM)
- Public transport rate
- Larger urban area resident population
- Population density (inh. /km2)

A rapid transit (RT) project corridor has a length (km) and a weekday average demand i.e. a weekday average mileage (trip km). The corridor subtracting a share of traffic by the road:

ΔRT = Road subtracted trip = RT demand · (1-Public transport rate)

On safe side:

ΔRM = Road subtracted mileage (trip km)
= Road subtracted trip · corridor length

Finally, [7], the average daily PM10 concentration reduction is:

$$\Delta PM10 = \varepsilon_R \cdot \frac{R_M}{\Delta R_M} \cdot \gamma$$

Then act on the distribution of trips, means acting on the circadian cycles of the city, improving global urban energy consumption efficiency and thus reducing pollutant emissions.

Conclusion

The prevailing model of development of urban areas is increasingly oriented to a territorial isotropy, notwithstanding the evidences of a considerable literature. This is due to the absence of mathematical models able to translate this awareness into appropriate financial models.

Nowadays a city no longer exists as mere physical built place, but as a dematerialized "cloud" of flows: people, goods, information. The vicious circle is all here: the car has allowed the urban sprawl; mass transit does not allow the widespread use of a city.

This is the time of urban sustainability. If each urban element (home-work-facilities) is framed dynamically within the daily circadian rhythm that connects them, it will be possible to develop a sustainable solution that is not partialized on a specific sector. A balanced city produces social wealth: that turns into a financial one because it can be measured.

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